Exercise 1. *Consider a triangle with its inscribed circle. Join each vertex of the triangle with the point of tangency of the opposite side with the inscribed circle. Prove that three lines you constructed pass through one point.*

In what proportion the point of intersection divides these three segments? Express your answer in terms of lengths of sides of the original triangle a; b; c.

Exercise 2. *Take a circle and points* A, B, C *and* P *on it. Take triangle* ABC. *Consider orthogonal projections* C_1, A_1, B_1 *of the point* P *on the line* AB, BC *and* CA *respectively. Prove that points* A_1, B_1, C_1 *belong to one line.*

Exercise 3. *Consider 3 circles. For every couple of circles consider their two common external tangent lines and take their point of intersection. Prove that these 3 points of intersection belong to one line.*

Exercise 4. *Prove that for any triangle three points of intersections of bisectors of its external angles with opposite sides belong to one line.*

Exercise 5. *Let* A', B', C' *be points on the sides* BC, CA, AB *of triangle* ABC *so that* $AC' : C'B = 3 : 2$, $BA' : A'C = 1 : 3$ *and* $AB' : B'A = 2 : 1$. *Let* E *be the point of intersection of* CC' *and* $A'B'$. *Find the ratio* $A'E : EB'$.

Exercise 6. *Consider a triangle ABC. Take points* A_1, B_1, C_1 *such that* $A_1 \in BC$ *and* $BA_1 : A_1C = 2 : 1$, $B_1 \in CA$ *and* $CB_1 : B_1A = 2 : 1$, $C_1 \in AB$ *and* $AC_1 : C_1B = 2 : 1$. *Consider the triangle cut by lines* AA_1, BB_1, CC_1. *Prove that its area is equal to* $\frac{1}{7}$ *of the area of the triangle ABC.*