

Exercise 1. Consider a tetrahedron. Join each vertex with the point of intersection of the medians of the opposite face. Prove that all 4 segments you constructed pass through one point. Prove that this point divides each segment in proportion 1: 3.

Exercise 2. Consider a 4-gon $ABCD$. Let K, L, M, N be the middle points of the segments AB, BC, CD, DA respectively. Let O be the point of intersection of the lines LN, KM . Let P and Q be the middle points of the diagonals AC and BD respectively. Show that O is the middle point of the segment PQ .

Exercise 3. Take a 6-gon $A_1A_2A_3A_4A_5A_6$. Let $B_1, B_2, B_3, B_4, B_5, B_6$ be the middle points of the sides $A_1A_2, A_2A_3, A_3A_4, A_4A_5, A_5A_6, A_6A_1$ respectively. Prove that the points of intersection of medians of triangles $B_1B_3B_5$ and $B_2B_4B_6$ coincide.

Exercise 4. Every vertex of a triangle was connected by two lines to the points that divide the opposite side to three equal parts. Prove that in the hexagon these six lines form the lines connecting opposite vertices are concurrent.

Exercise 5. Let O be the point of intersections of the medians of the triangle ABC . Let M, N, P divide the sides AB, BC, CA of the triangle ABC in equal proportions. Prove that O is also the point of intersection of the medians of triangle MNP .

Exercise 6. Let M, N, P be points on the sides AB, BC, CA of triangle ABC . Let M', N', P' be the reflections of the points M, N, P in the midpoints of sides AB, BC, CA . Prove that the areas of triangles MNP and $M'N'P'$ are equal.