Exercise 1. *Let ABC be a triangle and let E be an ellipse tangent to sides AB, BC, CA at points C', A', B' respectively. Show that AA', BB', CC' are concurrent.*

Exercise 2. *Suppose that an affine transformation sends a circle C to itself. Show that it sends the center of C to itself.*

Exercise 3. *Let E be an ellipse intersecting triangle ABC in six points $A_1, A_2, B_1, B_2, C_1, C_2$ so that the points A_1, A_2 are on the segment BC, points B_1, B_2 are on the segment CA, points C_1, C_2 are on the segment AB. Prove that if AA_1, BB_1, CC_1 are concurrent, then AA_2, BB_2, CC_2 are also concurrent.*

Exercise 4. *Let a, b, c be three vectors in the plane with the property that $\alpha a + \beta b + \gamma c = 0$ for some scalars α, β, γ. Prove that these vectors can be transformed to unit length vectors by an affine transformation if and only if there exists a triangle with side lengths $|\alpha|, |\beta|, |\gamma|$.*

Exercise 5. *Let E be an ellipse and let A, B be distinct points on it. Show that there are exactly two affine transformation that send E to itself and send the point A to point B.*

Exercise 6. *Given an acute triangle ABC prove that the perimeter of the triangle $A'B'C'$ whose vertices are the feet of the altitudes of ABC is less than twice the length of any height.*

Exercise 7. *Let ABC be a regular triangle ($AB = BC = CA$). Show that for every point P inside the triangle the sum of distances from P to the sides AB, BC and CA is the same.*

Exercise 8.

1. *Let l be a line and let A and B be two points lying on different sides of it. Assume that the quantity $|PA - PB|$ is largest at point P among all points P on L. Let Q be a point on l that is different from Q. Prove that $\angle APQ = \angle BPQ$*

2. *Use part 1 to explain why for a tangent line l to hyperbola given by the relation $PA - PB = const$" the following holds: if Q is a point on l distinct from P then $\angle APQ = \angle BPQ$*

Exercise 9. *Consider a triangle ABC. Assume that angles at the vertices A, B are smaller than $45°$. Let P be a point inside the triangle. Give an explicit construction that produces points $A' \in BC$, $B' \in CA$, $C' \in AB$ such that the sum $PA' + A'B' + B'C' + C'P$ is the smallest possible.*

Exercise 10. *Let ABC be a triangle with angles $\angle A = 30^\circ, \angle B = 60^\circ, \angle C = 90^\circ$. For what points P inside the triangle is the sum $PA' + \sqrt{3}PB' + PC'$ the largest, where A', B', C' are orthogonal projections of point P onto the sides BC, CA, AB respectively. For what points P inside the triangle it is the smallest?*