

THE FACULTY OF ARTS AND SCIENCE
University of Toronto

FINAL EXAMINATIONS, APRIL/MAY 2008

MAT402H CLASSICAL PLANE GEOMETRIES
AND THEIR TRANSFORMATIONS

Examiner: Professor A. Khovanskii

Duration: 3 hours

NO AIDS ALLOWED.

Total Marks: 100

1. [20 marks] Consider a triangle ABC . Let D be a middle of the side AB , and let E be a middle of the median CD . In what proportion a line AE divides the side CB ?
Hint: Put appropriate masses at the points A , B and C .
2. [20 marks] Take a circle S_0 and its diameter D . Take a chain of circles S_1, S_2, S_3, \dots such that circle S_1 is tangent to S_0 and is tangent to the diameter D at the center O ; the circle S_2 is tangent to S_0 , to D and to S_1 ; the circle S_3 is tangent to S_0 , to D and to S_2 and so on. Let A_1, A_2, \dots be the sequence of points of tangency of the circles S_1 and S_2 ; the circles S_2 and S_3 and so on. Prove there exists a circle S which contains all the points A_1, A_2, \dots
3. [20 marks] Consider a regular triangle ABC . Find all points O for which the sum $O_{AB} + 2O_{BC} + 3O_{CA}$ is the smallest possible. Here O_{AB}, O_{BC} and O_{CA} are distances from point O to the sides AB, BC and CA respectively.
4. [20 marks] Assume that for four lines a, b, c, d passing through a point P the cross-ratio (a, b, c, d) equals -1 . Prove: is the ray c bisects the angle between a and b , then d is perpendicular to c .
5. [20 marks] Prove converse of Desargues's theorem: if three points of intersections of the corresponding sides of two triangles ABC and $A'B'C'$ belong to one line then the lines joining corresponding vertices of the triangles pass through one point.
Hint: Apply arguments we used to prove Desargues's theorem.