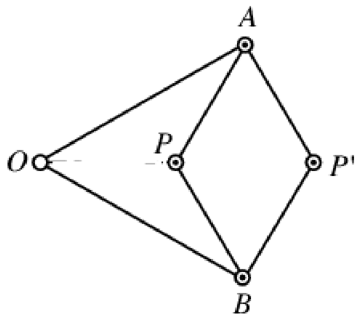
Problems that already appeared in preparation for the midterm:

- Term test 2008, problem 5;
- Is there an inversion that takes the points $(2,0),(-2,0),(0,2),(0,-1)$ into vertices of a square?
- For three given lines in the plane find a point O such that after an inversion centered at O the three lines become circles of equal radii.
- Describe the image under inversion of the family of lines passing through the point $(0,1)$ after inversion in the unit circle centered at the origin.
- Consider a circle inscribed in a square. Describe the image of the square after inversion in this circle.

Other problems:

- Let S be a circle with center O having radius r. Let P be a point outside the circle. Let l_1, l_2 be two tangent lines to the circle S passing through the point P. Let Q_1, Q_2 be the points of tangency of these lines with the circle. Show that the point of intersection of the segments OP and Q_1Q_2 is the image of P under inversion in S.
- Explain the action of Peaucellier's cell (see picture), an instrument for drawing the inverse of any given locus. It is formed by four equal rods, hinged at the corners of a rhombus $APBP'$, and two equal (longer) rods connecting two opposite corners, A and B, to a fixed pivot O. When a pencil point is inserted at P' and a tracing point at P (or vice versa), and the latter is traced over the curve of given figure, the pencil point draws the inverse figure.



- Show that two tangent spheres get inverted into parallel planes.

- Let S', S'', S''' be three spheres all touching one another. Let $Q_1, Q_2, \ldots$ be a sequence of spheres touching one another successively and all touching S', S'', S'''. Show that Q_6 is touching Q_1, so that we have a ring of 6 spheres interlocked with the original ring of 3 (hint: invert in a sphere whose center is the point of contact of S' and S'').

- Consider plane $\mathbf{R}^2$ as complex plane. Consider the transformation $z \to \frac{z+1}{z-1}$. Which circles under this transformation will become lines?

- Take two circles S_1 and S_2 intersecting at points A and B. Consider all circles S orthogonal to S_1 and to S_2. Find the locus of centers of all such circles S.

  Hint: How the points A and B are located with respect to a circle S?

- Let T_1 and T_2 be the inversions in the circles $x^2+y^2 = 16$ and $(x-8)^2+y^2 = 1$ respectively. Consider the composition W for these inversions $W = T_2 \circ T_1$. Which lines under the transformation W will become lines?

- Consider two non-concentric circles S_1 and S_2, one inside another. Assume that there exists a chain of circles $S_3, \ldots, S_{2005}$, such that each circle in the chain is tangent to the circles S_1 and S_2, and also to the next circle (i.e. S_3 is tangent to S_4, S_4 is tangent to S_5 and so on), and S_{2005} is tangent to S_3. Prove that for any other chain of circles $S'_3, \ldots, S'_{2005}$, such that each circle in the chain is tangent to the circles S_1 and S_2, and also to the next circle, the last circle S'_{2005} will be also tangent to S'_3.

  Hint: Using an inversion reduce the problem to a simpler form.

- Take a circle S_0 and its diameter D. Take a chain of circles $S_1, S_2, S_3, \ldots$ such that circle S_1 is tangent to S_0 and is tangent to the diameter D at the center O; the circle S_2 is tangent to S_0, to D and to S_1; the circle S_3 is tangent to S_0, to D and to S_2 and so on. Let $A_1, A_2, \ldots$ be the sequence of points of tangency of the circles S_1 and S_2, the circles S_2 and S_3 and so on. Prove that there exists a circle S which contains all the points $A_1, A_2, \ldots$

- Consider triangle ABC such that $AB = 3, BC = 4, CA = 5$. Find the point O such that after an invertion centered at O the line passing through A, C becomes a line, and lines passing through A, B and through B, C become circles of equal radii.

- Take two intersecting circles $S_1, S2$ on plane. Prove that there is a Moebius transformation which maps S_1, S_2 to two circles of the same radius.
- Take two non-intersecting circles $S_1, S2$ on plane. Prove that there is a Moebius transformation which maps S_1, S_2 to two concentric circles.