- Let P and Q be two distinct lines. Let $ABCDEF$ be a 6-gon (may be self-intersecting) such that the vertices A, C, E lie on the line P, while the vertices B, D, F lie on the line Q. For each of three pairs of opposite sides (i.e. AB and DE, BC and EF, CD and FA) take its intersection point. Prove that these three points belong to one line. (This theorem known as Pappus's theorem, is a degenerate case of Pascal's theorem when a conic section degenerates into a couple of lines P and Q; while proving Pappus's theorem try to follow the line of arguments we used in class when we were proving Pascal's theorem.)

- Let P and Q be two distinct points. Let $ABCDEF$ be a 6-gon (may be self-intersecting) such that the lines containing the edges A, C, E pass through the point P, while the lines containing the edges B, D, F pass through the point Q. For each of three pairs of opposite vertices (for example, intersection of sides A, B and intersection of sides D, E) take the line that joins them ("long diagonal of a 6-gon"). Prove that the three "long diagonals" pass through one point. (This theorem is dual to Pappus's theorem; it is a degenerate case of Brianchon's theorem and is known under the same name; while proving this theorem try to follow the line of arguments we used in class when we were proving Brianchon's theorem for nondegenerate conic sections.)

- Prove converse of Desargues's theorem: if three points of intersections of the corresponding sides of two triangles ABC and $A'B'C'$ belong to one line then the lines joining corresponding vertices of the triangles pass through one point. Hint: Apply arguments we used to prove Desargues's theorem.

- Take a sphere of radius R and a (spherical) triangle on this sphere with angles α, β, γ. Write a formula for the area of the triangle (in terms of the angles α, β, γ) and prove this formula.

- Consider a sphere S of radius R. Is it possible to locate 50 equal triangles with angles equal to $\pi/2, \pi/3, \pi/4$ on it in such a way that any two triangles do not overlap each other?