UNIVERSITY of TORONTO

MAT402H CLASSICAL PLANE GEOMETRIES

AND THEIR TRANSFORMATIONS

March 4, 2010 (from 6:10 to 9:00 p.m.)

Term test

No aids allowed.

Problem 1 (20 points).

Consider a triangle ABC. Assume that $\cos\alpha = \cos\beta = 1/4$, where α and β are angles at A and B. Find all points O in the triangle for which the sum $2O_{AB}+2O_{BC}+O_{CA}$ is the biggest possible. Here O_{AB}, O_{BC} and O_{CA} are distances from point O to the sides AB, BC and CA respectively.

Problem 2 (20 points).

Consider triangle ABC. Let D be a point on the side AB such that $AD : DB = 10$. Let E be a point on the segment CD such that $CE : ED = 11$. Let F be the point of intersection of the line L passing through AE and the side CB. Find $CF : FB$.

Hint: put appropriate masses at the vertices of the triangle in such a way that the point E becomes the center of masses.

Problem 3 (20 points). Let T_1 and T_2 be the inversions in the circles $x^2+y^2 = 16$ and $(x-8)^2+y^2 = 1$. Consider the composition W for these inversion $W = T_2 \circ T_1$. Which lines under the transformation W will become lines?

Problem 4 (20 points).

Consider a simple convex polyhedron Δ in R^3 with 2010 edges.

1) How many vertices are there in Δ?

2) How many faces are there in Δ?

Hint: Try do 1) by hands and then use the Euler characteristic formula for 2).

Problem 5 (20 points).

Prove the following theorem:

Let L be a line tangent at the point A to an ellipse with focuses O_1 and O_2. Then the rays AO_1 and AO_2 make equal angels with the line L.